

An Introduction to Geometric Analysis and it's Applications

Kevin R. Vixie

Los Alamos National Laboratory

September 8, 2004

An Introduction to Geometric Analysis and it's Applications

Kevin R. Vixie

Los Alamos National Laboratory

September 8, 2004

- Selim Esedoglu, UCLA

An Introduction to Geometric Analysis and it's Applications

Kevin R. Vixie

Los Alamos National Laboratory

September 8, 2004

- Selim Esedoglu, UCLA
- Tom Asaki CCS-2 (LANL), Rick Chartrand
T-7 (LANL),
- Bill Allard (Duke), David Caraballo (George-
town)

1. Functions, Derivatives, and Boundary values.
2. Details: why we need covering lemmas.
3. Application of trace results:
 L^1 TV exact solutions
4. Another Application:
 L^2 TV and Radiography

$L^1(\Omega)$ functions

$L^1(\Omega)$ functions

$$\{f \mid \int_{\Omega} |f| d\mu < \infty\} \quad (1)$$

Lebesgue's Theorem for $f \in L^1(\Omega)$

Lebesgue's Theorem for $f \in L^1(\Omega)$

$$\frac{\lim_{\rho \rightarrow 0} \int_{B_\rho(y)} |f(z) - f(y)| dz}{|B_\rho(y)|} = 0 \quad \text{a.e.} \quad (2)$$

What About $f(x)$ for $x \in \partial A$ for $A \subset \Omega$?

What About $f(x)$ for $x \in \partial A$ for $A \subset \Omega$?

on ∂A , a.e. should be H_{n-1} a.e. An example
(sequence of annuli converging to a circle).

How derivatives save us, enter $W^{1,1}$.

How derivatives save us, enter $W^{1,1}$.

$$W^{1,1} \equiv \{f | f \in L^1, Df \in L^1\} \quad (3)$$

where Df is a function such that

$$\int f \operatorname{div}(\vec{g}) = \int Df \cdot \vec{g} \quad (4)$$

for all smooth “test” functions g .

But is $W^{1,1}$ as general as possible?
 $\rightarrow BV(\Omega)$ ($|Df|$ is a radon measure).

But is $W^{1,1}$ as general as possible?
 $\rightarrow BV(\Omega)$ ($|Df|$ is a radon measure).

BV is the set of L^1 functions whose weak derivative is a Radon measure with finite variation.

Traces work for BV too!

Traces work for BV too!

Now we simply want to avoid discontinuities collecting on any mildly regular curve ∂A .

We want

$$\lim_{\rho \rightarrow 0} \rho^{-n} \int_{B_\rho(y)} |f(z) - Tf(y)| dz = 0 \quad (5)$$

for H_{n-1} almost all $y \in \partial A$.

We want

$$\lim_{\rho \rightarrow 0} \rho^{-n} \int_{B_\rho(y)} |f(z) - Tf(y)| dz = 0 \quad (6)$$

for H_{n-1} almost all $y \in \partial A$.

So we need

$$\rho^{-n} \int |f(z) - Tf(y)| \leq \rho^{1-n} \int_{C_\rho} |Df| + \rho^{1-n} \int_{D_\rho} |Tf(\eta) - Tf(y)| \quad (7)$$

Need to show Radon measures can't "collect" on ∂A .

Need to show Radon measures can't "collect" on ∂A .

$$\rho^{1-n} \int_{C_\rho} |Df| dz \quad (8)$$

or

$$\frac{1}{\rho^{n-1}} \int_{C_\rho} |Df| dz \quad (9)$$

We need a covering lemma to do this
→ Vitali covering lemma.

We need a covering lemma to do this
→ Vitali covering lemma.

Given $A \subseteq \mathbb{R}^n$, $\rho : A \rightarrow (0, 1)$ there exists a countable $\{x_i\} \in A$ such that

1. $B_{\rho(x_i)}(x_i) \cap B_{\rho(x_j)}(x_j) = \emptyset \quad i \neq j$
2. $A \subseteq \bigcup_{i=1}^{\infty} B_{3\rho(x_i)}(x_i)$

How we use this lemma: an outline.

How we use this lemma: an outline.

(On the Board!)

Brief result: *careful* proof with traces in BV .

Brief result: *careful* proof with traces in BV .

1. first: L^1 TV functional (compare: (L^2) TV functional).
2. exact solutions
3. $1/\lambda$ and $2/\lambda$ balls
4. $2/\lambda$ balls inside and outside: comparison using traces
5. (shrinking result and $1/\lambda$ results for convex Ω)

TV regularization to invert radiographs?

TV regularization to invert radiographs?

$$\min_u F(u) \equiv \int_{\Omega} |Df| + \lambda \int |P(u) - d|^2 \quad (10)$$

Four slides: Tom Asaki.

Four slides: Tom Asaki.

1. multiple view tomography
2. Abel projection inversions
→ (1 view tomography)
3. simulated and real data